13.1 | Faraday's Law

Learning Objectives

By the end of this section, you will be able to:

- Determine the magnetic flux through a surface, knowing the strength of the magnetic field, the surface area, and the angle between the normal to the surface and the magnetic field
- Use Faraday's law to determine the magnitude of induced emf in a closed loop due to changing magnetic flux through the loop

The first productive experiments concerning the effects of time-varying magnetic fields were performed by Michael Faraday in 1831. One of his early experiments is represented in **Figure 13.2**. An emf is induced when the magnetic field in the coil is changed by pushing a bar magnet into or out of the coil. Emfs of opposite signs are produced by motion in opposite directions, and the directions of emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.

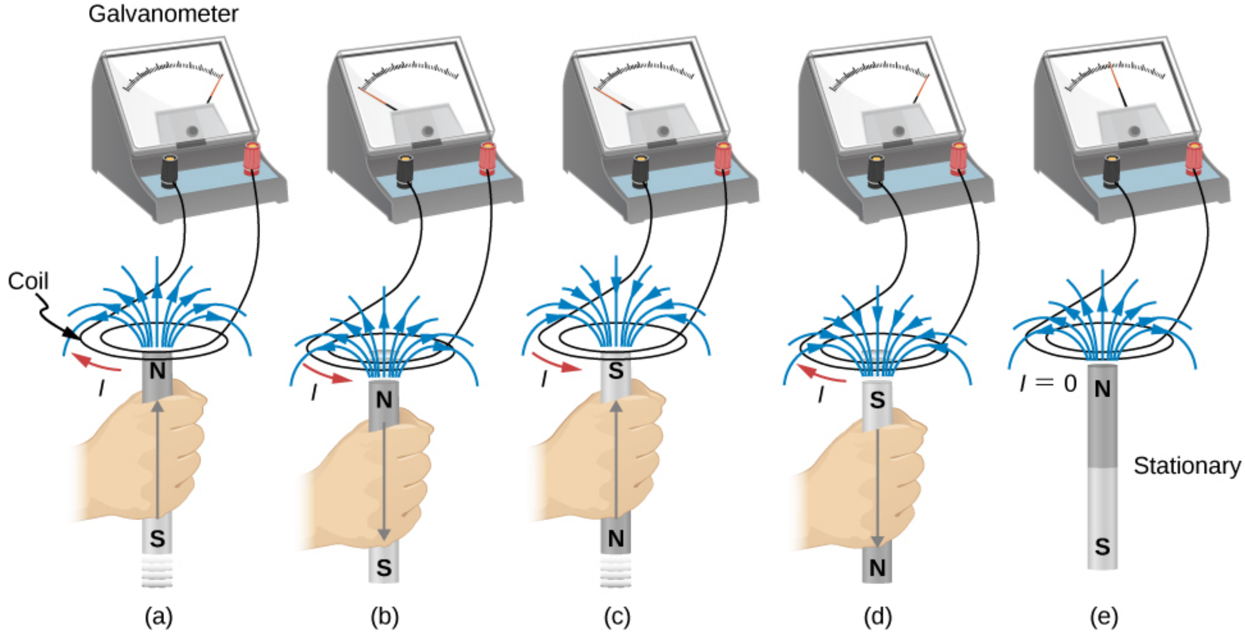

Figure 13.2 Movement of a magnet relative to a coil produces emfs as shown (a–d). The same emfs are produced if the coil is moved relative to the magnet. This short-lived emf is only present during the motion. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion, as shown in (e).

Faraday also discovered that a similar effect can be produced using two circuits—a changing current in one circuit induces a current in a second, nearby circuit. For example, when the switch is closed in circuit 1 of **Figure 13.3**(a), the ammeter needle of circuit 2 momentarily deflects, indicating that a short-lived current surge has been induced in that circuit. The ammeter needle quickly returns to its original position, where it remains. However, if the switch of circuit 1 is now suddenly opened, another short-lived current surge in the direction opposite from before is observed in circuit 2.

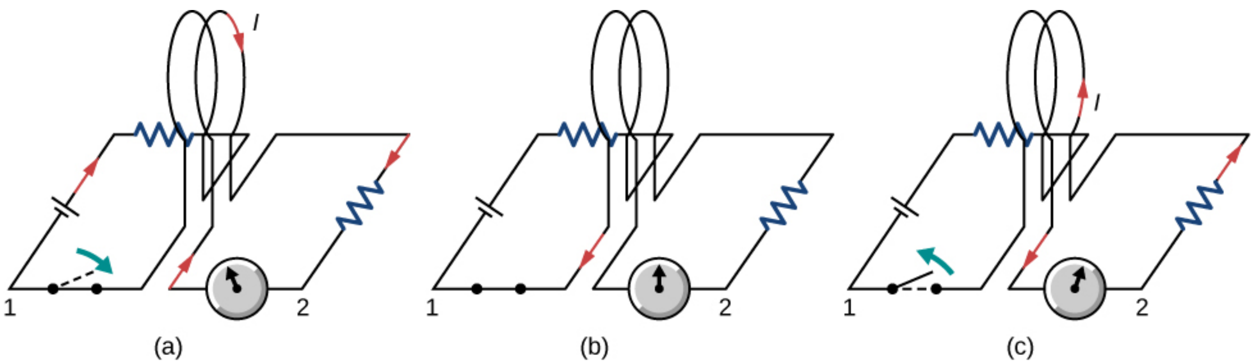

Figure 13.3 (a) Closing the switch of circuit 1 produces a short-lived current surge in circuit 2. (b) If the switch remains closed, no current is observed in circuit 2. (c) Opening the switch again produces a short-lived current in circuit 2 but in the opposite direction from before.

Faraday realized that in both experiments, a current flowed in the circuit containing the ammeter only when the magnetic field in the region occupied by that circuit was *changing*. As the magnet of the figure was moved, the strength of its magnetic field at the loop changed; and when the current in circuit 1 was turned on or off, the strength of its magnetic field at circuit 2 changed. Faraday was eventually able to interpret these and all other experiments involving magnetic fields that vary with time in terms of the following law:

Faraday's Law

The emf ε induced is the negative change in the magnetic flux Φ_m per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

The **magnetic flux** is a measurement of the amount of magnetic field lines through a given surface area, as seen in **Figure 13.4**. This definition is similar to the electric flux studied earlier. This means that if we have

$$\Phi_m = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, dA, \tag{13.1}$$

then the **induced emf** or the voltage generated by a conductor or coil moving in a magnetic field is

$$\varepsilon = -\frac{d}{dt}\int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, dA = -\frac{d\Phi_m}{dt}. \tag{13.2}$$

The negative sign describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with a rule known as Lenz's law, which we will discuss shortly.

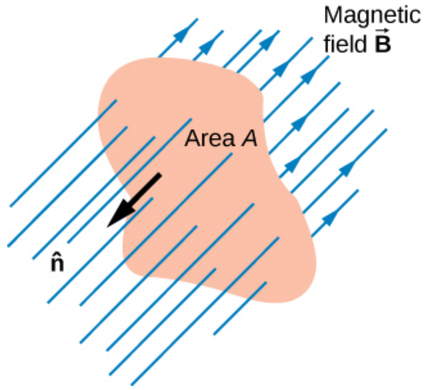

Figure 13.4 The magnetic flux is the amount of magnetic field lines cutting through a surface area A defined by the unit area vector $\hat{\mathbf{n}}$. If the angle between the unit area $\hat{\mathbf{n}}$ and magnetic field vector $\vec{\mathbf{B}}$ are parallel or antiparallel, as shown in the diagram, the magnetic flux is the highest possible value given the values of area and magnetic field.

Part (a) of **Figure 13.5** depicts a circuit and an arbitrary surface S that it bounds. Notice that S is an *open surface*. It can be shown that *any* open surface bounded by the circuit in question can be used to evaluate Φ_{m}. For example, Φ_{m} is the same for the various surfaces $S_1, S_2, \ldots$ of part (b) of the figure.

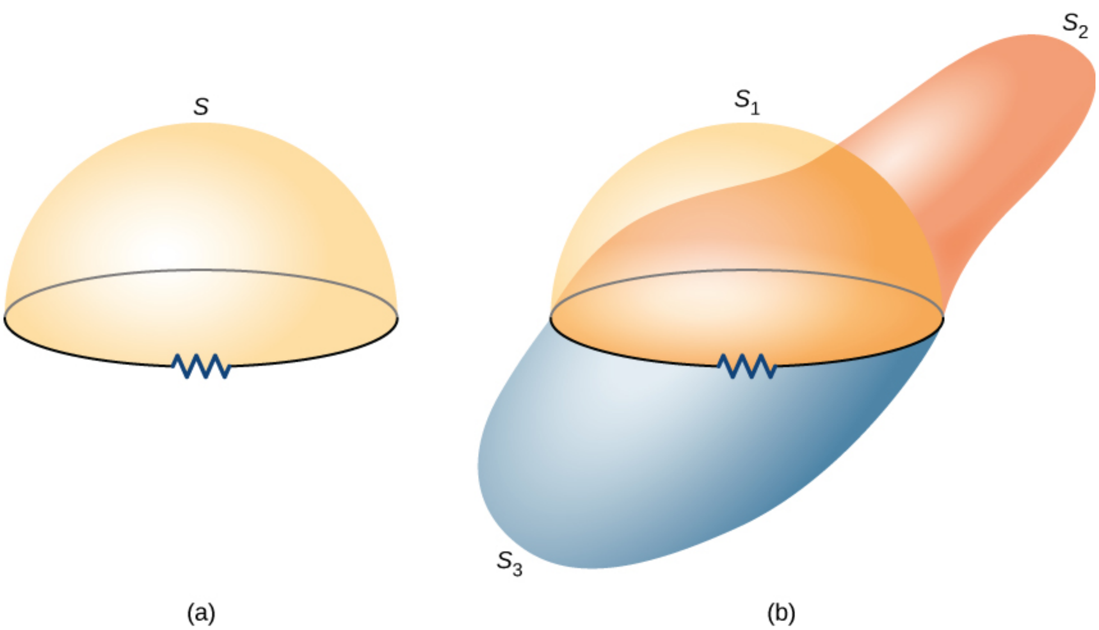

Figure 13.5 (a) A circuit bounding an arbitrary open surface S. The planar area bounded by the circuit is not part of S. (b) Three arbitrary open surfaces bounded by the same circuit. The value of Φ_{m} is the same for all these surfaces.

The SI unit for magnetic flux is the weber (Wb),

$$1\ \text{Wb} = 1\ \text{T} \cdot \text{m}^2.$$

Occasionally, the magnetic field unit is expressed as webers per square meter (Wb/m^2) instead of teslas, based on this definition. In many practical applications, the circuit of interest consists of a number N of tightly wound turns (see **Figure 13.6**). Each turn experiences the same magnetic flux. Therefore, the net magnetic flux through the circuits is N times the flux through one turn, and Faraday's law is written as

$$\varepsilon = -\frac{d}{dt}(N\Phi_m) = -N\frac{d\Phi_m}{dt}. \tag{13.3}$$

Example 13.1

A Square Coil in a Changing Magnetic Field

The square coil of **Figure 13.6** has sides $l = 0.25\ \text{m}$ long and is tightly wound with $N = 200$ turns of wire. The resistance of the coil is $R = 5.0\ \Omega$. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = -0.040\ \text{T/s}$. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

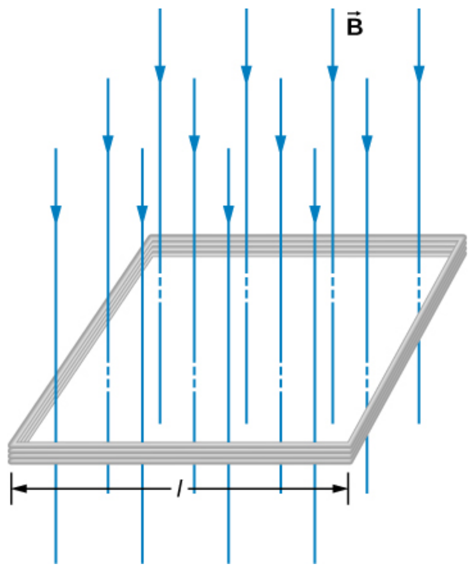

Figure 13.6 A square coil with N turns of wire with uniform magnetic field $\vec{\mathbf{B}}$ directed in the downward direction, perpendicular to the coil.

Strategy

The area vector, or $\hat{\mathbf{n}}$ direction, is perpendicular to area covering the loop. We will choose this to be pointing downward so that $\vec{\mathbf{B}}$ is parallel to $\hat{\mathbf{n}}$ and that the flux turns into multiplication of magnetic field times area. The area of the loop is not changing in time, so it can be factored out of the time derivative, leaving the magnetic field as the only quantity varying in time. Lastly, we can apply Ohm's law once we know the induced emf to find the current in the loop.

Solution

a. The flux through one turn is

$$\Phi_m = BA = Bl^2,$$

so we can calculate the magnitude of the emf from Faraday's law. The sign of the emf will be discussed in the next section, on Lenz's law:

$$\begin{aligned} |\varepsilon| &= \left|-N\frac{d\Phi_m}{dt}\right| = Nl^2\frac{dB}{dt} \\ &= (200)(0.25\ \text{m})^2(0.040\ \text{T/s}) = 0.50\ \text{V}. \end{aligned}$$

b. The magnitude of the current induced in the coil is

$$I = \frac{\varepsilon}{R} = \frac{0.50\ \text{V}}{5.0\ \Omega} = 0.10\ \text{A}.$$

Significance

If the area of the loop were changing in time, we would not be able to pull it out of the time derivative. Since the loop is a closed path, the result of this current would be a small amount of heating of the wires until the magnetic field stops changing. This may increase the area of the loop slightly as the wires are heated.

13.1 Check Your Understanding A closely wound coil has a radius of 4.0 cm, 50 turns, and a total resistance of $40\,\Omega$. At what rate must a magnetic field perpendicular to the face of the coil change in order to produce Joule heating in the coil at a rate of 2.0 mW?

13.2 | Lenz's Law

Learning Objectives

By the end of this section, you will be able to:

- Use Lenz's law to determine the direction of induced emf whenever a magnetic flux changes
- Use Faraday's law with Lenz's law to determine the induced emf in a coil and in a solenoid

The direction in which the induced emf drives current around a wire loop can be found through the negative sign. However, it is usually easier to determine this direction with **Lenz's law**, named in honor of its discoverer, Heinrich Lenz (1804–1865). (Faraday also discovered this law, independently of Lenz.) We state Lenz's law as follows:

Lenz's Law

The direction of the induced emf drives current around a wire loop to always *oppose* the change in magnetic flux that causes the emf.

Lenz's law can also be considered in terms of conservation of energy. If pushing a magnet into a coil causes current, the energy in that current must have come from somewhere. If the induced current causes a magnetic field opposing the increase in field of the magnet we pushed in, then the situation is clear. We pushed a magnet against a field and did work on the system, and that showed up as current. If it were not the case that the induced field opposes the change in the flux, the magnet would be pulled in produce a current without anything having done work. Electric potential energy would have been created, violating the conservation of energy.

To determine an induced emf ε, you first calculate the magnetic flux Φ_m and then obtain $d\Phi_m/dt$. The magnitude of ε is given by $\varepsilon = |d\Phi_m/dt|$. Finally, you can apply Lenz's law to determine the sense of ε. This will be developed through examples that illustrate the following problem-solving strategy.

Problem-Solving Strategy: Lenz's Law

To use Lenz's law to determine the directions of induced magnetic fields, currents, and emfs:

1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the applied magnetic field $\vec{\mathbf{B}}$.
3. Determine whether its magnetic flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field $\vec{\mathbf{B}}$. The induced magnetic field tries to reinforce a